V. *Addition to the Paper on "Volcanic Energy: an attempt to develop its true Origin and Cosmical Relations"* *. *By* Robert Mallet, *A.M., C.E., F.R.S., M.R.I.A.*

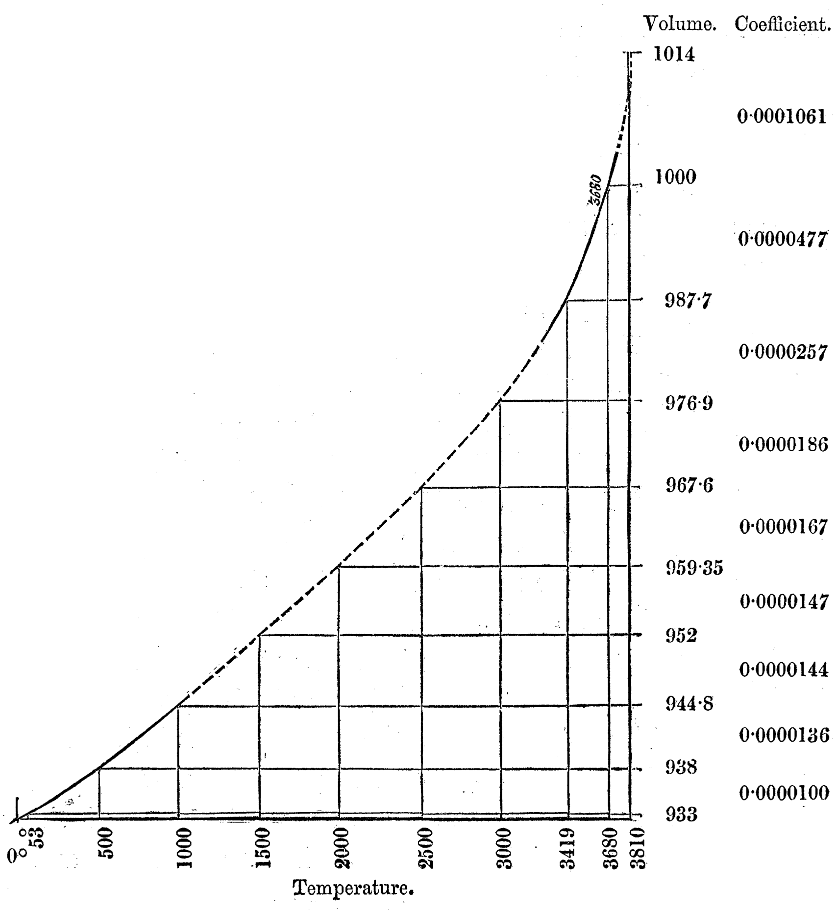
Received April 3,—Read May 7, 1874.

In the paper whose title is given above (Philosophical Transactions, part i. 1873) the author has shown upon experimental data, and upon the acknowledged basis that the amount of heat annually dissipated from our globe equals that evolved by 777 cubic miles of ice at 32° melted to water at the same temperature, what is the amount of heat that can be annually produced by the transformation of the mechanical work of mean rock when crushed by the descent of the external shell upon the nucleus contracting beneath it; he has also estimated the annual supply of heat necessary for the maintenance of the volcanic activity at present existing upon our globe; has shown that its total amount cannot exceed a small fraction of the entire heat dissipated annually, being only $\frac{1}{1589}$ thereof, or, in terms of crushed mean rock, equal 0·5579 of a cubic mile (paragraphs 179 and 197); he has also given, in Table II. (page 201) and succeeding paragraphs, his experimental results as to the contraction by diminution of temperature of melted matter that may be presumed similar to the rocky material of our globe from which natural lavas are derived. This contraction in volume, in relation to temperature between that of the blast-furnace and of the atmosphere, is shown graphically by the curve Plate x. of the above paper, the upper and lower portions of the curve being derived from experiment. The preceding elements afford some of the data necessary for any calculation as to the actual contraction of our globe now taking place annually by its secular refrigeration; but the author refrained from attempting any such calculation on the grounds that other data indispensable to any certain result are yet wanting. If we knew the thickness of the earth's solid shell and the true increment of hypogeal temperature from the surface to the centre, or even the mean temperature of the nucleus and the nature of the whole of the matter composing the latter, we might with some assurance approximate to the amount of annual contraction of the globe due to refrigeration. But of the deep interior of our planet we really *know* but two things, viz. that the interior is hotter than the exterior, and what is the mean density of the whole. By making certain suppositions, however, as to some of the chief data wanted, we may approximate to some probable measures of the present annual contraction, and be enabled to see how far the results tend to sustain or overthrow the views enunciated by the author as to the nature and origin of volcanic heat and energy, and may also find

* Read June 20, 1872; Philosophical Transactions for 1873, p. 147.

that they throw some additional light upon the conjectured thicknesses that have been assigned to the earth's solid crust, as well as upon the question left undecided by LAPLACE as to how far the effects of contraction due to refrigeration would be astronomically observable during the period of scientific history. In the author's paper above referred to he has only dealt with the total contraction of the slag experimented upon between the temperature of its issue from the blast-furnace (viz. 3680°) and that of the atmosphere (53°), or by volume from 1000 to 933 for 3617° FAHR., from which the Rev. O. FISHER has calculated a mean coefficient of contraction $=0\cdot0000217$ for 1° FAHR. (Geol. Mag., February 1874). This, though sufficient for that able writer's immediate object, is not quite correct, as it treats the curve of contraction (Plate x. Philosophical Transactions, 1873) as a straight line. And in order to make use for our present purpose of these experimental contractions, it is necessary to obtain partial mean coefficients for different portions of the entire curve. This the author has done for ranges of about 500° between the temperatures of the blast-furnace and that of the atmosphere. The diagram fig. 1 (reduced from Plate x. Philosophical Transactions, 1873) shows the

Fig. 1.—Curve of Total Contraction of Slags.



intervals of temperature within which the mean coefficients for contraction in volume have been calculated; the results are probably sufficiently clear on inspection, but may be tabulated thus:—

TABLE I.—Coefficient of Contraction of Slags experimented upon at Barrow.

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
Higher temperature Fahr.	Lower temperature Fahr.	Range of temperature.	Volume at higher temperature taken as	Volume at lower temperature then equals	Volume at 3680° Fahr. taken as 1000, then volume at other temperatures is as	Total contraction from volume at 3680° to volume at each following temperature.	Amount of contraction between each two temperatures.	Coefficients of contraction per degree Fahr.	Mean coefficient.
3810°	3680°	130°	1014	1000	1014	$\frac{14}{1014}$	$\frac{14}{1014}$	0·0001061	
3680	3419	261	1000	987·7	987·7	$\frac{12·3}{1000}$	$\frac{12·3}{1000}$	0·0000477	0·000020087 (rows 3680–500)
3419	3000	419	1000	989	976·9	$\frac{23·1}{1000}$	$\frac{10·8}{1000}$	0·0000257	
3000	2500	500	1000	991	967·6	$\frac{32·4}{1000}$	$\frac{9·3}{1000}$	0·0000186	
2500	2000	500	1000	992	959·35	$\frac{40·65}{1000}$	$\frac{8·35}{1000}$	0·0000167	
2000	1500	500	1000	993	952·00	$\frac{48}{1000}$	$\frac{7·5}{1000}$	0·0000147	
1500	1000	500	1000	993	944·80	$\frac{55·2}{1000}$	$\frac{7·2}{1000}$	0·0000144	
1000	500	500	1000	993	938·00	$\frac{62}{1000}$	$\frac{6·8}{1000}$	0·0000136	
500	53	447	1000	995	933·00	$\frac{67}{1000}$	$\frac{5}{1000}$	0·0000100	

From inspection of the diagram fig. 1 and Table I., the upper and lower portions of both of which (between 3680° and 53°) are reliable as being experimentally obtained, we may observe that the mean coefficient of contraction in volume for the total range of temperature shown in the diagram is =0·00002972 for one degree of FAHR. reduction in temperature, or to 0·000020087, or very nearly 0·0000201 for the limits of temperature actually embraced by experiment, being those employed by the Rev. O. FISHER. We also observe that the rate of dilatation or of contraction in volume for the two uppermost segments of the curve, viz. between the temperatures 3419° and 3810°, or a range of 391°, is 6·4 times greater than that for the two lowermost segments of the curve, viz. from 53° to 1000°, or a range of 947° FAHR. If, therefore, the mean temperature of the nucleus of our globe be assumed within the limits of the former, and that of the shell within those of the latter, and the capacity for heat of both the same, the contraction in volume of the former will be 6·4 times that of the latter for an equal decrement of temperature in both.

It is immaterial as to what follows whether we regard the nucleus of our globe as solid or liquid, or in what way or through what intermediate state of viscosity the solid shell may pass into the nucleus if the latter be liquid; it is only necessary for the author to postulate a higher temperature, and therefore a larger coefficient of contraction, for the interior of the globe than for the colder shell which surrounds it, and to suppose as was done by the late Mr. Hopkins in his researches as to the thickness of the shell in relation to precession, that, whatever thickness may be assigned to the shell, it passes *per saltum* into the nucleus—all that is here meant being, that all below this imaginary *couche* contracts more than does all above it for a given decrement of temperature of both. We have no certain knowledge of the rate at which temperature increases either in the shell or the nucleus in descending from the surface, nor what may be the highest temperature of the nucleus itself; but as the mean temperature of the shell may be presumed greatly inferior to that of the nucleus, it may be allowable to regard the whole of the heat dissipated from our globe in a unit of time (a year) as derived from the nucleus only, and transmitted merely through the shell, the thickness of the latter being taken as not too large in relation to the earth's radius. The total heat dissipated from our globe in a year, or, on the above suppositions, from the nucleus only, being, as above stated, equal to that evolved by the melting of 777 cubic miles of ice at 32° to water at 32°, may be considered for any moderate secular period, such as 5000 years, as constant. The refrigerative power of the unit of volume of a cubic foot of such ice is

$$C=\varrho\times s,$$

ϱ being the specific gravity and s the latent heat of ice. Therefore

$$C=57{\cdot}8\times143=8265^{\circ}{\cdot}4 \text{ Fahr.},$$

or units of heat, assuming the capacity for heat of water to be the same at all temperatures; and the refrigerative effect of this upon an equal volume of the mass of the nucleus is

$$\frac{C}{s'\times\varrho'},$$

s' and ϱ' being the specific heat and specific gravity or weight per unit of volume, respectively, of the matter of the nucleus. We in reality know nothing as to what may be the chemical or physical nature of the matter composing the nucleus; we therefore have no basis for assigning its specific heat in whole or in part; nor do we know any thing as to its specific gravity beyond this, that the mean density of our globe being 5·5, that of the nucleus alone must be somewhat greater. We are therefore obliged to adopt the most probable suppositions we can for the values of s' and ϱ'. It is highly probable, as appears to be generally conjectured by geologists, that a large proportion at least of the entire mass of our globe, and therefore of the nucleus as here defined, consists of rocky material not very dissimilar from that known to us by observation or inference in the superficial crust of the earth. Now as none of the materials of the crust, excluding those of metallic veins or beds relatively small in quantity, at all approach the average density of 5·5, we may reasonably conclude that towards the centre of our planet there exist masses of metals, the only bodies we are acquainted with whose high specific gravities would bring the mean density of the whole to 5·5. The exterior portions of the sphere, constituting by far the largest portion of its entire volume, have a density of little more than 2·0. But we cannot deal with the absolutely unknown, nor assign either specific heat or specific gravity to the extremely dense material, whether metallic or not, which we must suppose to exist about the centre of figure of our planet. The most reasonable supposition, therefore, that we can make in reference to our present object is to neglect the nature of this extremely dense matter, and to assume the whole nucleus as composed of material not greatly different from the hardest and densest rocks with which we are acquainted, and, with some allowance for their further increase in density by compression, to adopt for the whole nucleus a value for ϱ', a density of 2·75 (or one half the mean density of our entire globe), and for its specific heat $s'=0{\cdot}200$, being a little above the mean experimentally ascertained by the author for the five hardest and densest rocks in Table I. column 27 of his paper in Philosophical Transactions, 1873. The equation

$$\frac{\mathrm{C}}{s'\times\varrho'}$$

therefore becomes

$$\frac{8265{\cdot}4}{0{\cdot}2\times2{\cdot}75\times62{\cdot}425}=24^{\circ}{\cdot}74 \text{ Fahr.},$$

which is the amount of refrigeration produced by a unit in volume (1 cubic foot) of melted ice upon an equal volume of the nucleus. Having for the constant refrigerative power the 777 cubic miles of melted ice, and having the volume of the nucleus for any assigned thickness of shell, we at once obtain the amount of refrigeration of the nucleus; and applying to that the partial mean coefficient of contraction for 1° Fahr. found at the upper portions of our curve, we are enabled to calculate the reduction in volume, and hence the diminution in radius, due to the amount of heat abstracted in the unit of time, viz. one year. The author has assumed four successive thicknesses for the shell, viz.

$$\left.\begin{array}{r}100\\200\\400\\800\end{array}\right\}\text{ miles,}$$

and proceeding on the above principles has calculated the total annual contraction of the nucleus for each case. The partial mean coefficient of contraction adopted for that of the nucleus has been the mean between the two highest partial means shown in the curve and Table I. above given, viz. 0·0000769 for 1° Fahr.

The final results obtained are comprised in Table II., before referring to which, however, some explanation and reference to diagram fig. 2 are necessary.

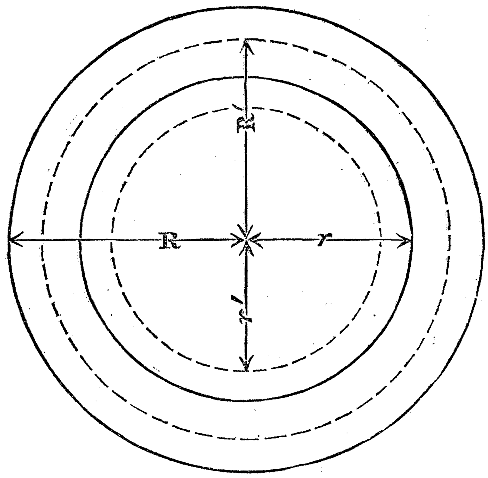
R being the radius of our globe$=3957{\cdot}5$ English miles,

$r=$the radius assumed for the nucleus, whose thickness $=\mathrm{R}-r$.

Let the nucleus be assumed to contract by loss of its heat transmitted through the shell until its radius $=r'$, the shell then, in following down after the contracted nucleus, must descend everywhere through a vertical height equal $r-r'$.

The spherical shell having the original external and internal radii R and r must

Fig. 2.



accommodate itself to this descent so as to remain in contact with the diminished nucleus: it may do this in either of two ways; it may increase in thickness, or $R'-r'$ be greater than $R-r$; or the thickness $R-r$ may remain constant, in which case, as the volume of the shell after descent is less than before, a certain portion of its volume must be extruded or got rid of in some way. In the earlier stages of our globe's refrigeration, as explained in the author's paper of 1873, the thickness of the descending shell did not remain constant, but was increased by external corrugations and wrinklings, and other like changes due to tangential pressure in that epoch of mountain-raising. But the epoch of mountain-building has practically ceased, the shell being too thick and rigid to admit of it. The thickness of the shell now must therefore be viewed as constant, and the accommodation of its volume to enable it to remain in contact with the contracting nucleus is produced by extrusion of some of its mass blown out to the surface by volcanic action. The difference in volume thus to be got rid of is the difference between

$$n\{(2R)^3-(2r)^3\} \text{ and } n\{(2R')^3-(2r')^3\},$$

$$\text{the constant } n=\frac{\pi}{6} \text{ being}=\cdot 5236,$$

and as stated, the thickness of the shell remaining constant, the thickness of the imaginary spherical shell which measures the reduction in volume of the nucleus, or $r-r'$, must be $=$ to the vertical descent of the external surface of the original or uncontracted shell, or

$$r-r'=R-R';$$

and as the absolute thickness of both these imaginary spherical shells is small, the quantity of matter to be extruded is proportionate to the difference between their internal or external surfaces respectively, or as

$$(2r')^2 : (2R')^2.$$

In dealing with these enormous volumes, this relation affords a convenient method of determining the volume of matter that must be extruded from the shell.

TABLE II.

1.	2.	3.	4.	5.	6.	7.	8.
Thickness of shell.	Diameter of nucleus.	Volume of nucleus.	Volume of contracted nucleus.	Reduction in volume due to heat lost.	Radial contraction in miles.	Volume of extruded matter.	Radial contraction in inches.
miles. 100	miles. 7715	240440392958·15	240440392956·6793	1·470602	0·000000007806	0·07663643	0·0004945
200	7515	222211764992·15	222211764990·67939		0·00000000836	0·15374	0·0005296
400	7115	188592463139·15	188592463137·67939		0·00000000928	0·34636	0·00058995
800	6315	131882013356·15	131882013354·67939		0·0000000106	0·758199	0·0006716

The results arrived at are seen at one view in Table II. On examining the Table, it will be seen that the diminution in volume of the nucleus is constant whatever be the thickness of the shell, for the obvious reason that the absolute reduction in temperature, and therefore the absolute contraction in volume, are inversely as the mass of the nucleus acted upon by the constant refrigeration, 777 cubic miles of melted ice; but the radial contraction is greater as the volume of the nucleus is smaller. Recalling from the author's paper of 1873 the result that 0·5579 of a cubic mile of crushed mean rock is the amount annually necessary for the maintenance of the volcanic activity of our globe at present (an amount which the author believes to exceed the actual truth), and viewing such crushed rock as the same thing with the extruded matter of the shell, it will be seen that, on the suppositions we have made, the thickness of the solid shell of our globe necessary for the support of its volcanic activity must exceed 400 miles, and that with a thickness of shell of 800 miles the annual volume of the extruded or crushed rock exceeds by about one half the quantity required to support volcanic activity. As the rigid shell is and has been for ages in a state of elastic compression by tangential thrusts, it is easily perceived that any increase, however slowly taking place, in these compressive strains must be promptly responded to by disturbances in the mechanical equilibrium of the shell itself. Some minute portion of these may perhaps still be disposed of in small partial thickenings of the shell itself, giving rise to slight secular variations in level, such as have been observed in Scandinavia and Greenland; but these expiring remains of ancient mountain-building are relatively so minute that they may be disregarded here. The reliability of the conclusions here arrived at is of course only proportionate to the admissibility of the suppositions made upon which they depend. In so far, however they tend to support the author's views as to the nature and origin of volcanic heat and energy, and also to support the views of those who regard the solid crust of our globe as necessarily much thicker than geologists generally have been in the habit of admitting it. It is probable that the contractions here determined for our planet are below the truth; for

1st. Some contraction must always take place through cooling of the solid shell itself, and especially of its lowermost and hottest portions, which has been here neglected.

2nd. It is probable that the coefficient of contraction employed is below the truth for the material of the nucleus such as we have supposed it.

If the central parts of the nucleus be metallic, it is probable that their coefficient of contraction may largely exceed that here employed, while their specific heat is considerably less than that adopted for the entire nucleus. On the other hand, it must be remembered that a wave of heat from the central parts of the nucleus may take ages to travel conductively outwards to the lower surface of the shell, even when the latter is assumed 800 miles in thickness, which is one of the reasons why in what precedes these central parts have been supposed of a nature similar to the nucleus. It follows that, on the supposition of a shell of 800 miles in thickness, the annual diminution in diameter of our globe, due to its secular refrigeration, may somewhat exceed, but cannot be less than, $\frac{1}{1493396226414}$ of its diameter, a mere film wholly incapable of being recognized by the senses; or taking the diminution of diameter from the unit of a British inch instead of a mile, it would amount in a period of 5000 years to a diminution of the diameter of our globe of 6·71616 inches, or less than 7 inches, a quantity so small that it must have escaped the most refined observation of the astronomer during the last 2000 years, even were we to suppose that during the whole of that period the instrumental resources of the astronomer were as perfect as at the present day. When we add to this the consideration that the matter composing the imaginary spherical shell of less than $3\frac{1}{2}$ inches in thickness, which measures the contraction in volume of our globe during 5000 years, has by its refrigeration increased in density in the ratio at least of 1000 to 933, we readily discern the reasons for the negative results arrived at by LAPLACE in considering this question from the point of view of an observable diminution in the length of the day. Yet insignificant when thus measured as is the amount of annual contraction of our globe by its secular refrigeration, we see how important and mighty are its effects in preserving through the volcano the cosmical regimen of our world; it is another added to the many instances already known in the range of natural philosophy, in which causes so minute as for long to remain occult to us are yet, though unseen and unnoticed, essential parts of the mighty machine.

Three quantities related to each other indeed, but yet entirely different, have been treated of in the author's present paper or in that of 1873.

1st. The volume of mean rock which must be crushed annually in the earth's shell in order to supply the heat necessary for existing annual vulcanicity, viz. 0·5579 of a cubic mile, the heat due to which is $\frac{1}{1589}$ of the total annually dissipated from our globe.

2ndly. The volume of matter that must be annually crushed and extruded from the shell of 800 miles in thickness in order to admit of its following down after the contracting nucleus, being 0·758199 of a cubic mile, which, if measured in terms of mean crushed rock, amounts to $\frac{1}{1176}$ of the heat annually dissipated. The former of these quantities is comprised within the second as its source of supply, which, as we observe, exceeds the annual demand necessary for existing volcanic energy by about one half.

3rdly. The volume of material heated or molten annually blown out of all the volcanic vents of our globe, as based upon the estimates made in the author's original paper (paragraphs 195 to 197), which amounts to 0·1486 of a cubic mile, a quantity probably in excess of the truth. The first of these quantities, upon the data assumed in this paper, would be produced by a thickness of the solid shell of our earth of more than 400 but less than 800 miles. The third of those quantities might be accounted for by a shell of more than 200, but less than 400 miles in thickness. If the shell be actually less than the smallest of these thicknesses, it follows either that the annual dissipation of heat from our globe greatly exceeds that due to 777 cubic miles of melted ice, or that the coefficient of contraction for the nucleus here employed and based on experiment is below the truth, neither of which suppositions is improbable. It will be remarked that the results in this paper have been obtained by an independent and different method of investigation from that employed in the author's original paper (Philosophical Transactions of 1873), and that they coordinate to such an extent as to support the probability of the truth of the views enunciated in both papers.

In conclusion I wish to acknowledge the efficient aid I have received from my assistant Mr. W. Worby Beaumont, Assoc. Inst. C.E., in completing the laborious calculations involving a mass of figures of which the results only are here seen.